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Comments on holographic gravity dual of $\mathcal{N} = 6$ superconformal Chern-Simons gauge theory

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ABSTRACT: The holographic nonsupersymmetric renormalization group flows in four dimensions are found. The mass-deformed $\mathcal{N} = 2, 4$ Chern-Simons matter theories can be reproduced from $\mathcal{N} = 1$ Chern-Simons matter theory by putting some constraints in the mass terms. We construct the geometric superpotential, from an eleven dimensional M-theory lift, which provides M2-brane probe analysis for the infrared ends of various supersymmetric or nonsupersymmetric flows.

KEYWORDS: AdS-CFT Correspondence, M-Theory

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1 Introduction

An explicit construction of the renormalization group(RG) flow between the ultraviolet(UV) fixed point and the infrared(IR) fixed point of the three dimensional field theory has a close relation to a supergravity kink solution in four dimensions. There exist holographic RG flow equations connecting $\mathcal{N} = 8$ SO(8) fixed point to $\mathcal{N} = 2$ SU(3) \times U(1) fixed point [1, 2]. Moreover, the other holographic RG flow equations from $\mathcal{N} = 8$ SO(8) fixed point to $\mathcal{N} = 1$ G_2 fixed point also exist [2, 3]. The exact solutions to the eleven-dimensional bosonic equations corresponding to the M -theory lift of these RG flows are known in [3, 4].

The three dimensional $\mathcal{N} = 6$ Chern-Simons matter theories with gauge group $U(N) \times U(N)$ and level k have been constructed in [5] and this theory is described as the low energy limit of N M2-branes at $\mathbf{C}^4/\mathbf{Z}_k$ singularity. The mass deformed $U(2) \times U(2)$ Chern-Simons gauge theory with level $k = 1$ or $k = 2$ which preserves SU(3) \times U(1) symmetry is studied in [6-8]. For G_2 symmetry case, the corresponding mass deformation is described in [9].

Besides the above two supersymmetric critical points, there exist also three nontrivial nonsupersymmetric critical points for the scalar potential:SO(7)⁺, SO(7)⁻ and SU(4)⁻. Although there were some partial attempts in [2] for the finding of RG flow equations interpolating $\mathcal{N} = 8$ SO(8) fixed point to these nonsupersymmetric fixed points, we are going to further describe those RG flow equations behind those nonsupersymmetric fixed points. In order to see the whole structure of the six critical points including the maximal supersymmetric SO(8) case, we need to use the SU(3)-invariant sectors(i.e., the SU(3) is a common subgroup of above six invariant groups) of gauged $\mathcal{N} = 8$ supergravity in four dimensions. Moreover, the eleven-dimensional metric for the whole SU(3)-invariant sector [10], realized as a warped product of an asymptotically AdS_4 space with a squashed and stretched seven-sphere, is crucial for M2-brane probe.

In this paper, starting from the first order differential equations, that are the nonsupersymmetric flow solutions in four dimensional $\mathcal{N} = 8$ gauged supergravity interpolating between an exterior AdS_4 region with maximal $\mathcal{N} = 8$ supersymmetry and an interior AdS_4 with no supersymmetry, we would like to interpret this as the RG flow in Chern-Simons matter theory broken to the deformed Chern-Simons matter theory by the addition of mass terms for the adjoint superfields. An exact correspondence can be obtained between fields of bulk supergravity in the AdS_4 region in four-dimensions and composite operators of the IR field theory in three-dimensions. The three dimensional analog of Leigh-Strassler [11] RG flow in mass-deformed Chern-Simons matter theory in three dimensions is expected. We present the results of probing the eleven-dimensional supergravity solution corresponding to RG flows.

In section 2, we review the $SU(3)$ -invariant supergravity solutions in four dimensions in the context of RG flow, describe the various supergravity critical points found previously and present the new nonsupersymmetric flow equations.

In section 3, the $\mathcal{N} = 2$ $SU(3) \times U(1)$ -invariant bosonic mass terms in the Lagrangian can be reproduced by putting the constraints for the mass terms to the more generic $\mathcal{N} = 1$ G_2 -invariant bosonic mass terms. Moreover, $\mathcal{N} = 4$ $[SU(2) \times U(1)]^2$ -invariant bosonic deformed Lagrangian can be constructed. We also study $\mathcal{N} = \frac{1}{2}$ and nonsupersymmetric ($\mathcal{N} = 0$) mass-deformed theories.

In section 4, the eleven-dimensional geometric superpotential which reduces to the usual AdS_4 superpotential for the particular internal coordinates is described for the M2-brane analysis of moduli space. In particular, the IR behaviors at the nonsupersymmetric critical points are emphasized.

In section 5, the future directions are presented.

2 The holographic RG flows for $SU(3)$ -invariant sector in four dimensions

The gauged $\mathcal{N} = 8$ theory contains self-interaction of a single massless $\mathcal{N} = 8$ supermultiplet of spins $(2, \frac{3}{2}, 1, \frac{1}{2}, 0^+, 0^-)$ with local $SO(8)$ and local $SU(8)$ invariance. In particular, there exists a non-trivial effective potential for the scalars that is proportional to the square of the $SO(8)$ gauge coupling g . The 70 real, physical scalars characterized by $(0^+, 0^-)$ of $\mathcal{N} = 8$ supergravity parametrize the coset space $E_{7(7)}/SU(8)$ since 63 fields may be gauged away by an $SU(8)$ rotation. Then they are described by an element $\mathcal{V}(x)$ of the fundamental 56-dimensional representation of $E_{7(7)}$:

$$\mathcal{V}(x) = \begin{pmatrix} u_{ij}^{IJ}(x) & v_{ijKL}(x) \\ v^{klIJ}(x) & u^{kl}_{KL}(x) \end{pmatrix},$$

where $SU(8)$ index pairs $[ij], \dots$ and $SO(8)$ index pairs $[IJ], \dots$ are antisymmetrized and u_{ij}^{IJ} and v_{ijKL} fields are 28×28 matrices. Any ground state leaving the symmetry unbroken is necessarily AdS_4 space with a cosmological constant proportional to g^2 . Although the full gauged $\mathcal{N} = 8$ Lagrangian is rather complicated, the scalar and gravity part of the action is simple. Let us define $SU(8)$ T-tensor which is manifestly antisymmetric in the

indices $[ij]$ and SU(8) covariant:

$$T_l^{kij} = \left(u_{IJ}^{ij} + v^{ijIJ} \right) \left(u_{lm}^{JK} u_{KI}^{km} - v_{lmJK} v^{kmKL} \right).$$

This occurs naturally by introducing a local gauge coupling in the theory. Furthermore, other tensors coming from T-tensor play an important role and they appear in the g -dependent interaction terms. That is, A_1 tensor is symmetric in (ij) and A_2 tensor is antisymmetric in $[ijk]$:

$$A_1^{ij} = -\frac{4}{21} T_m^{ijm}, \quad A_{2l}^{ijk} = -\frac{4}{3} T_l^{[ijk]}.$$

The former appears in the variation of the gravitino of the theory while the latter appears in the variation of 56 Majorana spinor of the theory. The SO(8) of gauged supergravity acts on \mathbf{R}^8 as the vector representation and there exists SU(3) subgroup leaving all the four-forms invariant where there are three self-dual four-forms and three anti-self-dual four-forms. Although there exist six scalar fields in $\mathcal{N} = 8$ supergravity due to the six four-forms, only four of them are parametrized as the SU(3) singlet space and they live in the submanifold of $E_{7(7)}/\text{SU}(8)$ after using the two residual U(1)'s in SO(8) and putting two four-forms to be zero.

The SU(3)-invariant sector of the scalar manifold of gauged $\mathcal{N} = 8$ supergravity [12] in four dimensions has been studied in [2]. The critical points of scalar potential have led to AdS_4 vacua and the SU(3) gauge symmetry in the supergravity side is preserved. Then the SU(3)-invariant scalar potential of gauged $\mathcal{N} = 8$ supergravity in terms of original variables [13] can be written as

$$V(\lambda, \lambda', \alpha, \phi) = g^2 \left[\frac{16}{3} \left| \frac{\partial z_3}{\partial \lambda} \right|^2 + 4 \left| \frac{\partial z_3}{\partial \lambda'} \right|^2 - 6 |z_3|^2 \right] \quad (2.1)$$

where the z_3 is the one of the eigenvalues of A_1 tensor of the theory

$$z_3(\lambda, \lambda', \alpha, \phi) = 6e^{i(\alpha+2\phi)} p^2 q r^2 t^2 + 6e^{2i(\alpha+\phi)} p q^2 r^2 t^2 + p^3 (r^4 + e^{4i\phi} t^4) + e^{3i\alpha} q^3 (r^4 + e^{4i\phi} t^4). \quad (2.2)$$

Here we introduce various hyperbolic functions as follows:

$$p \equiv \cosh \left(\frac{\lambda}{2\sqrt{2}} \right), \quad q \equiv \sinh \left(\frac{\lambda}{2\sqrt{2}} \right), \quad r \equiv \cosh \left(\frac{\lambda'}{2\sqrt{2}} \right), \quad t \equiv \sinh \left(\frac{\lambda'}{2\sqrt{2}} \right). \quad (2.3)$$

Although the equation (2.1) doesn't contain the derivative terms with respect to the fields α and ϕ , one can write down the scalar potential in terms of "true" superpotential W , by using the algebraic relations found in [2] between the complex function z_3 and its derivatives with respect to the fields $\lambda, \lambda', \alpha$ and ϕ ,

$$V(\lambda, \lambda', \alpha, \phi) = g^2 \left[\frac{16}{3} (\partial_\lambda W)^2 + \frac{2}{3p^2 q^2} (\partial_\alpha W)^2 + 4 (\partial_{\lambda'} W)^2 + \frac{1}{2r^2 t^2} (\partial_\phi W)^2 - 6W^2 \right]$$

with superpotential W which is the magnitude of complex function z_3 (2.2):

$$W(\lambda, \lambda', \alpha, \phi) = |z_3|. \quad (2.4)$$

There exist six critical points of this scalar potential. Three of them are supersymmetric while the other three are nonsupersymmetric. The symmetry group has a common SU(3) group.

SO(8) critical point. This occurs at $\lambda = 0 = \lambda'$, the cosmological constant is $\Lambda = -6g^2$ (and $W = 1$) and the $\mathcal{N} = 8$ supersymmetry is preserved.

SO(7)⁺ critical point. This occurs at $\lambda = \sqrt{2} \sinh^{-1}(\sqrt{\frac{1}{2}(\frac{3}{\sqrt{5}} - 1)}) = \lambda'$ and $\alpha = 0 = \phi$. There is no supersymmetry and the cosmological constant is given by $\Lambda = -2 \cdot 5^{\frac{3}{4}} g^2$ (and $W = \frac{3}{2} \cdot 5^{-\frac{1}{8}}$).

SO(7)⁻ critical point. This occurs at $\lambda = \sqrt{2} \sinh^{-1}(\frac{1}{2}) = \lambda'$ and $\alpha = \frac{\pi}{2} = \phi$. There is no supersymmetry and the cosmological constant is given by $\Lambda = -\frac{25\sqrt{5}}{8} g^2$ (and $W = \frac{3}{8} \cdot 5^{\frac{3}{4}}$).

G₂ critical point. There is a critical point at $\lambda = \sqrt{2} \sinh^{-1}(\sqrt{\frac{2}{5}(\sqrt{3} - 1)}) = \lambda'$ and $\alpha = \cos^{-1}(\frac{1}{2}\sqrt{3 - \sqrt{3}}) = \phi$ and the cosmological constant is $\Lambda = -\frac{216\sqrt{2}}{25\sqrt{5}} \cdot 3^{\frac{1}{4}} g^2$ (and $W = \sqrt{\frac{36\sqrt{2} \cdot 3^{\frac{1}{4}}}{25\sqrt{5}}}$). This has an unbroken $\mathcal{N} = 1$ supersymmetry.

SU(4)⁻ critical point. This occurs at $\lambda = 0, \lambda' = \sqrt{2} \sinh^{-1}(1)$ and $\phi = \frac{\pi}{2}$. There is no supersymmetry and the cosmological constant is given by $\Lambda = -8g^2$ (and $W = \frac{3}{2}$).

SU(3) × U(1) critical point. Finally, there is a critical point at $\lambda = \sqrt{2} \sinh^{-1}(\frac{1}{\sqrt{3}}), \lambda' = \sqrt{2} \sinh^{-1}(\frac{1}{\sqrt{2}}), \alpha = 0$ and $\phi = \frac{\pi}{2}$ and the cosmological constant is $\Lambda = -\frac{9\sqrt{3}}{2} g^2$ (and $W = \frac{3^{\frac{3}{4}}}{2}$). This critical point [14] has an unbroken $\mathcal{N} = 2$ supersymmetry.

For the supergravity description of the nonconformal RG flow from one scale to another connecting any two critical points, the four-dimensional metric which contains three dimensional Poincare invariant metric has the form $ds^2 = e^{2A(r)} \eta_{\mu'\nu'} dx^{\mu'} dx^{\nu'} + dr^2$ where $\eta_{\mu'\nu'} = (-, +, +)$ and r is the coordinate transverse to the domain wall. Then the supersymmetric flow equations with (2.4) and (2.3) are described as [2]

$$\begin{aligned}
 \frac{d\lambda}{dr} &= \frac{8\sqrt{2}}{3} g \partial_\lambda W, & \frac{d\lambda'}{dr} &= 2\sqrt{2} g \partial_{\lambda'} W, \\
 \frac{d\alpha}{dr} &= \frac{\sqrt{2}}{3p^2 q^2} g \partial_\alpha W, & \frac{d\phi}{dr} &= \frac{\sqrt{2}}{3r^2 t^2} g \partial_\phi W, \\
 \frac{dA}{dr} &= -\sqrt{2} g W.
 \end{aligned} \tag{2.5}$$

When we restrict to the case of G_2 symmetry, the supersymmetric G_2 invariant reduced flow equations from SO(8) to G_2 [3] can be described also. On the other hand, for the SU(3) × U(1) symmetry case, the supersymmetric reduced flow equations from SO(8) to SU(3) × U(1) [1] are described similarly.

Although the nonsupersymmetric flow equations from SO(8) to SO(7)⁺ in the context of SO(5) invariant sector are found in [2], one can easily rederive them in the present context: SU(3) invariant sector. By substituting the domain-wall ansatz metric into the

Lagrangian of scalar-gravity sector, the Euler-Lagrangian equations are given in terms of the functional $E[A, \lambda]$. Then the energy-density per unit area transverse to r -direction is given by

$$E[A, \lambda] = \int_{-\infty}^{\infty} dr e^{3A} \left[-3 \left((\partial_r A)^2 + \partial_r^2 A \right) - \frac{7}{8} (\partial_r \lambda)^2 - V(\lambda) \right].$$

One can rewrite this as the sum of complete squares plus others using the squaring procedure. One arrives at

$$E[A, \lambda] = \int_{-\infty}^{\infty} dr e^{3A} \left[3 \left(\partial_r A + \sqrt{2} g W_+ \right)^2 - \frac{7}{8} \left(\partial_r \lambda - \frac{8\sqrt{2}}{7} g \partial_\lambda W_+ \right)^2 \right] - 2\sqrt{2} (e^{3A} W_+) |_{-\infty}^{\infty}.$$

Then the functional $E[A, \lambda]$ is extremized by the domain-wall solutions. The first order differential equations, the gradient nonsupersymmetric flow (we'll explain this further later) from $SO(8)$ to $SO(7)^+$, are written as

$$\frac{d\lambda}{dr} = \frac{8\sqrt{2}}{7} g \partial_\lambda W_+, \quad \frac{dA}{dr} = -\sqrt{2} g W_+ \quad (2.6)$$

where the superpotential W_+ can be obtained from the generic one W (2.4) by inserting $\alpha = 0 = \phi$ and $\lambda = \lambda'$. The scalar potential is given by $V(\lambda) = g^2 \left[\frac{16}{7} (\partial_\lambda W_+)^2 - 6W_+^2 \right]$. At the supersymmetric $SO(8)$ fixed point, $\partial_\lambda W_+$ vanishes while at the nonsupersymmetric $SO(7)^+$ fixed point, it doesn't vanish.

Similarly, the nonsupersymmetric flow from $SO(8)$ to $SO(7)^-$ can be expressed as

$$\frac{d\lambda}{dr} = \frac{8\sqrt{2}}{7} g \partial_\lambda |W_-|, \quad \frac{dA}{dr} = -\sqrt{2} g |W_-| \quad (2.7)$$

where the complex superpotential W_- can be obtained from the generic complex function z_3 (2.2) by inserting $\alpha = \frac{\pi}{2} = \phi$ and $\lambda = \lambda'$ and the scalar potential is given by $V(\lambda) = g^2 \left[\frac{16}{7} |\partial_\lambda W_-|^2 - 6|W_-|^2 \right]$. At the supersymmetric $SO(8)$ fixed point, $\partial_\lambda |W_-|$ vanishes while at the nonsupersymmetric $SO(7)^-$ fixed point, it doesn't vanish.

Compared with the two supersymmetric flows which have two independent fields, the nonsupersymmetric flows (2.6) and (2.7) have only one independent field $\lambda(r)$ in addition to the scale function $A(r)$. Once we choose either $SO(7)^+$ or $SO(7)^-$, then both α and ϕ are fixed automatically and only $\lambda(r)$ is left.

As far as the energy functional procedure we described before is concerned, there is no difference between the supersymmetric flows and nonsupersymmetric flows. We need to check whether they are supersymmetric or nonsupersymmetric flows by using other method. It is known in [15] that the domain wall solution for which the scalar is strictly monotonic is supersymmetric for the superpotential. The domain wall solutions that are asymptotic to unstable AdS_4 vacua (that violate Breitenlohner-Freedman bound) are nonsupersymmetric.

One can easily check that the above first order differential equations (2.6) and (2.7) satisfy the gravitational and scalar equations of motion by second order differential equations: $4\partial_r^2 A + 6(\partial_r A)^2 + \frac{7}{4}(\partial_r \lambda)^2 + 2V = 0$ and $\partial_r^2 \lambda + 3(\partial_r A)(\partial_r \lambda) - \frac{4}{7}\partial_\lambda V = 0$. By differentiating the scalar potential V , one obtains $\partial_\lambda V = 4g^2(\partial_\lambda W)(\frac{8}{7}\partial_\lambda^2 W - 3W)$ where

W is W_+ or $|W_-|$. At the critical point of V , one has $\partial_\lambda W = 0$ or $\frac{8}{7}\partial_\lambda^2 W = 3W$. The AdS_4 vacua arising from the former condition $\partial_\lambda W = 0$ (these are also critical points of W) are supersymmetric and stable by supersymmetry while those arising from the latter condition $\frac{8}{7}\partial_\lambda^2 W = 3W$ are nonsupersymmetric. The $SO(7)^\pm$ critical points hold for this latter condition as we mentioned before.

According to the observation from [16] in old 80's, the two critical points $SO(7)^\pm$ are unstable against small fluctuations corresponding to the **27** representations of $SO(7)$ because the Breitenlohner-Freedman bound is violated. See [16] for detailed computations on the stability for other representations of $SO(7)$. The question how a stable domain wall solution can be asymptotic to an unstable AdS_4 vacuum is answered in [15] in the general context of accumulation point of $\partial_r \lambda$.

Is there any nonsupersymmetric flow from $SO(8)$ to $SU(4)^-$? In this case, the scalar potential is given by $V(\lambda') = g^2 [8 (\partial_{\lambda'} W_-)^2 - 8W_-^2 + 2]$ where W_- can be obtained when we put $\lambda = 0$ and $\phi = \frac{\pi}{2}$ into the superpotential W . Then, one cannot make the energy density as the sum of complete square as well as others due to the last term of $V(\lambda')$.

We'll come back these flow equations (2.5), (2.6) or (2.7) when we describe the behavior of moduli space around IR fixed points in section 4.

3 The (non)supersymmetric membrane flows in three dimensions

Let us recall that the self-dual and anti self-dual tensors are given by, in $\mathcal{N} = 8$ gauged supergravity,

$$\begin{aligned} Y_{ijkl}^{1\pm} &= \varepsilon_\pm [(\delta_{ijkl}^{1234} \pm \delta_{ijkl}^{5678}) + (\delta_{ijkl}^{1256} \pm \delta_{ijkl}^{3478}) + (\delta_{ijkl}^{3456} \pm \delta_{ijkl}^{1278})], \\ Y_{ijkl}^{2\pm} &= \varepsilon_\pm [-(\delta_{ijkl}^{1357} \pm \delta_{ijkl}^{2468}) + (\delta_{ijkl}^{2457} \pm \delta_{ijkl}^{1368}) + (\delta_{ijkl}^{2367} \pm \delta_{ijkl}^{1458}) + (\delta_{ijkl}^{1467} \pm \delta_{ijkl}^{2358})], \end{aligned} \tag{3.1}$$

where $\varepsilon_+ = 1$ and $\varepsilon_- = i$ and $+$ gives the scalars while $-$ gives the pseudo-scalars. The indices $ijkl$ refer to $SU(8)$ indices but after gauge fixing there is no difference between $SU(8)$ indices and $SO(8)$ indices. The $SU(3)$ singlet space that is invariant subspace under a particular $SU(3)$ subgroup of $SO(8)$ has a linear combination of these four anti-symmetric tensors with four fields $\lambda, \lambda', \alpha$ and ϕ appeared in previous section. For the G_2 symmetric case, one should turn on all these four tensors $Y_{ijkl}^{1\pm}$ and $Y_{ijkl}^{2\pm}$ while for the $SU(3) \times U(1)$ symmetric case, one should turn on only Y_{ijkl}^{1+} and Y_{ijkl}^{2-} . This indicates that one expects the mass deformation in boundary theory, preserving the latter, can be obtained from the mass deformation preserving the former by adding some constraints on the mass parameters. We'll illustrate this explicitly.

Let us start with the fermionic mass terms of BL theory [17]:

$$\begin{aligned} \mathcal{L}_{f.m.} &= -\frac{i}{2} h_{ab} \bar{\Psi}^a (m_1 \Gamma^{78910} + m_2 \Gamma^{56910} - m_3 \Gamma^{5678} + m_4 \Gamma^{46810} \\ &\quad + m_5 \Gamma^{4679} + m_6 \Gamma^{4589} - m_7 \Gamma^{45710} - m_8 \mathbf{1}) \Psi^b. \end{aligned} \tag{3.2}$$

The indices in eleven-dimensional Gamma matrices correspond to the self-dual tensor (or anti self-dual tensor) for the indices 5678, 3478, 3456, 2468, 2457, 2367, 2358, if we shift all

the indices by adding 2 to (3.1), besides an identity. The corresponding fermionic supersymmetric transformation due to the mass deformation is given by

$$\delta_m \Psi^a = (m_1 \Gamma^{78910} + m_2 \Gamma^{56910} - m_3 \Gamma^{5678} + m_4 \Gamma^{46810} + m_5 \Gamma^{4679} + m_6 \Gamma^{4589} - m_7 \Gamma^{45710} - m_8 \mathbf{1}) X_I^a \Gamma_I \epsilon. \quad (3.3)$$

Let us classify the possible mass deformations according to the number of supersymmetry as follows.

$\mathcal{N} = 1$ supersymmetry. As done in [9](See also [18]), the $\frac{1}{8}$ BPS condition(the number of supersymmetry is two) requires the following constraints on the ϵ supersymmetry parameter

$$\Gamma^{5678} \epsilon = \Gamma^{56910} \epsilon = \Gamma^{78910} \epsilon = \Gamma^{46810} \epsilon = -\Gamma^{4679} \epsilon = -\Gamma^{4589} \epsilon = -\Gamma^{45710} \epsilon = -\epsilon. \quad (3.4)$$

One can easily check this BPS condition by constructing the eleven-dimensional Gamma matrices explicitly.

By taking the equal mass condition

$$m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = m_8 = m,$$

the bosonic mass term preserving $\mathcal{N} = 1$ supersymmetry

$$\mathcal{L}_{b.m.} = -\frac{1}{2} h_{ab} X_I^a (m^2)_{IJ} X_J^b \quad (3.5)$$

has the following result which has only one nonzero component

$$(m^2)_{IJ} = \text{diag}(0, 0, 0, 0, 0, 0, 0, 64m^2).$$

After integrating out the massive scalar field at low energy scale, the sixth order superpotential arises [9].

Motivated by the fact that the two M2-branes theory of BL theory is equivalent to $U(2) \times U(2)$ Chern-Simons matter theory with level $k = 1$ or $k = 2$ (there is further enhancement from $\mathcal{N} = 6$ to $\mathcal{N} = 8$ supersymmetry), the natural question is to ask what happens for Chern-Simons matter theory when we turn on mass perturbation in the gauged supergravity? Let us consider the $U(2) \times U(2)$ Chern-Simons matter theory. The theory has matter multiplet in seven flavors Φ_i where $i = 1, \dots, 7$ transforming in the adjoint with flavor symmetry G_2 under which the matter multiplet forms a septet $\mathbf{7}$ of the $\mathcal{N} = 1$ theory. Now we turn on the mass perturbation in the UV and flow to the IR. This maps to turning on certain fields in the AdS_4 supergravity. By integrating out the massive scalar Φ_8 that is a singlet $\mathbf{1}$ of G_2 with adjoint index, this results in the 6-th order superpotential $\text{Tr}(Y^{+ijk8} \Phi_i \Phi_j \Phi_k)^2 + \text{Tr} \epsilon_{ijklmnp} Y^{+ijk8} (\Phi_l \Phi_m \Phi_n \Phi_p)$ [9]. Thus we have found $\mathcal{N} = 1$ superconformal Chern-Simons matter theories with global G_2 symmetry and the Chern-Simons matter theories with G_2 -invariant superpotential deformation are dual to the holographic RG flows in [10]. We expect that G_2 -invariant $U(N) \times U(N)$ Chern-Simons matter theory for $N > 2$ with $k = 1, 2$ where there exists an enhancement of $\mathcal{N} = 8$ supersymmetry [5, 7] is dual to the background of [10] with N unit of flux. It would be interesting to explore this direction further.

$\mathcal{N} = 2$ supersymmetry. We impose the constraints on the ϵ parameter that satisfies the $\frac{1}{4}$ BPS condition (the number of supersymmetries is four):

$$\Gamma^{5678}\epsilon = \Gamma^{56910}\epsilon = \Gamma^{78910}\epsilon = -\epsilon. \quad (3.6)$$

Let us further impose the following conditions

$$m_1 = m_2 = m_3 = m_8 = 0. \quad (3.7)$$

Using the supersymmetry variation for X_I^a , $\delta X_I^a = i\bar{\epsilon}\Gamma_I\Psi^a$, and the supersymmetry variation for Ψ^a by the equation (3.3) due to the mass deformation, the variation for the bosonic mass term (3.5) plus the fermionic mass term (3.2) under the constraints (3.7) leads to

$$\delta\mathcal{L} = ih_{ab}X_I^a(m^2)_{IJ}\bar{\Psi}^b\Gamma_J\epsilon - ih_{ab}\bar{\Psi}^a(m_4\Gamma^{46810} + m_5\Gamma^{4679} + m_6\Gamma^{4589} - m_7\Gamma^{45710})^2 X_I^b\Gamma_I\epsilon.$$

Then the bosonic mass term $(m^2)_{IJ}\Gamma_J$ should take the form

$$\begin{aligned} (m^2)_{IJ}\Gamma^J \rightarrow & (m_4 - m_5 - m_6 + m_7)^2(\Gamma_3 + \Gamma_4) + (m_4 - m_5 + m_6 - m_7)^2(\Gamma_5 + \Gamma_6) \\ & + (m_4 + m_5 - m_6 - m_7)^2(\Gamma_7 + \Gamma_8) + (m_4 + m_5 + m_6 + m_7)^2(\Gamma_9 + \Gamma_{10}) \end{aligned} \quad (3.8)$$

where (3.6) are used. When all the mass parameters are equal $m_4 = m_5 = m_6 = m_7 = m$, then the diagonal bosonic mass term in (3.8) has nonzero component only for 99 and 1010 and other components (33, 44, 55, 66, 77, 88) are vanishing. The degeneracy 2 is related to the $\mathcal{N} = 2$ supersymmetry. Then one obtains the bosonic mass term which appears in (3.5)

$$(m^2)_{IJ} = \text{diag}(0, 0, 0, 0, 0, 0, 16m^2, 16m^2).$$

After integrating out the massive scalar field at low energy scale, the sixth order superpotential occurs [6].

The mass deformed BL theory with two M2-branes is equivalent to the mass deformed $U(2) \times U(2)$ Chern-Simons gauge theory of [5] with level $k = 1$ or $k = 2$. The theory has matter multiplet in three flavors Φ_i where $i = 1, 2, 3$ transforming in the adjoint with flavor symmetry $SU(3)_I$. The $SO(8)_R$ symmetry of the $\mathcal{N} = 8$ gauge theory is broken to $SU(3)_I \times U(1)_R$ where the former is a flavor symmetry under which the matter multiplet forms a triplet and the latter is the R-symmetry of the $\mathcal{N} = 2$ theory. Therefore, we turn on the mass perturbation in the UV and flow to the IR. This maps to turning on certain scalar fields in the AdS_4 supergravity. We can integrate out the massive scalar Φ_4 that is a singlet $\mathbf{1}$ of $SU(3)_I$ with adjoint index at a low enough scale and this results in the 6-th order superpotential $\text{Tr}(f_{abc}f^{ABCD}\Phi_A^a\Phi_B^b\Phi_C^c)^2$ [6]. The $SU(3)_I \times U(1)_R$ -invariant $U(N) \times U(N)$ Chern-Simons matter theory for $N > 2$ with $k = 1, 2$ is an open problem.

In this way, the $\mathcal{N} = 2$ superconformal Chern-Simons matter theory by adding mass terms can be constructed from $\mathcal{N} = 1$ superconformal Chern-Simons matter theory by relaxing the constraints on some of the mass terms. See also the recent work [19] where the $\mathcal{N} = 1$ supersymmetric RG flow from G_2 symmetric point to the $SU(3)_I \times U(1)_R$ symmetric point in BL theory. Now let us continue to study more supersymmetric case.

$\mathcal{N} = 4$ supersymmetry. We impose the constraint on the ϵ parameter that satisfies the $\frac{1}{2}$ BPS condition (the number of supersymmetries is eight) [20]: $\Gamma^{78910}\epsilon = -\epsilon$. We impose the additional constraints on m_4 and m_5 as well as previous ones (3.7):

$$m_1 = m_2 = m_3 = m_8 = 0, \quad \text{and} \quad m_4 = m_5 = 0.$$

The variation for the bosonic mass term plus the fermionic mass term leads to

$$\delta\mathcal{L} = ih_{ab}X_I^a(m^2)_{IJ}\bar{\Psi}^b\Gamma_J\epsilon - ih_{ab}\bar{\Psi}^a(m_6\Gamma^{4589} - m_7\Gamma^{45710})^2 X_I^b\Gamma_I\epsilon.$$

Then the bosonic mass term $(m^2)_{IJ}\Gamma_J$ should take the form

$$(m^2)_{IJ}\Gamma^J \rightarrow (m_6 - m_7)^2(\Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6) + (m_6 + m_7)^2(\Gamma_7 + \Gamma_8 + \Gamma_9 + \Gamma_{10}).$$

When the two mass parameters are equal $m_6 = m_7 = m$, then the diagonal bosonic mass term has nonzero component only for (77, 88, 99 and 1010) and other components (33, 44, 55, 66) are vanishing. The degeneracy 4 is related to the $\mathcal{N} = 4$ supersymmetry. Then one obtains the bosonic mass term which appears in (3.5)

$$(m^2)_{IJ} = \text{diag}(0, 0, 0, 0, 4m^2, 4m^2, 4m^2, 4m^2).$$

In this way, the $\mathcal{N} = 4$ superconformal Chern-Simons matter theory by adding mass terms can be constructed from $\mathcal{N} = 1$ (or $\mathcal{N} = 2$) superconformal Chern-Simons matter theory by relaxing the constraints on some of the mass terms. According to the classification for the critical points in previous section, there is no $\mathcal{N} = 4$ supersymmetric critical point. So this theory might be dual to the $\mathcal{N} = 4$ $[\text{SU}(2) \times \text{U}(1)]^2$ invariant gauged supergravity found in [21]. Or one should understand the $\text{SO}(3)$ (or $\text{SU}(2)$) invariant sector of gauged $\mathcal{N} = 8$ supergravity. We expect the full superpotential $\frac{M}{2}\text{Tr}(\Phi_3)^2 + \frac{M}{2}\text{Tr}(\Phi_4)^2 + \epsilon^{AB}\epsilon^{\hat{C}\hat{D}}\text{Tr}(\Phi_A\Phi_B\Phi_{\hat{C}}\Phi_{\hat{D}})$, where $\Phi_A (A = 1, 3)$ is $\mathbf{2}$ representation of one $\text{SU}(2)$ and $\Phi_{\hat{C}} (\hat{C} = \hat{2}, \hat{4})$ is $\mathbf{2}$ representation of the other $\text{SU}(2)$, in terms of $\mathcal{N} = 2$ superfields. See also recent work on $\mathcal{N} = 4$ superfield formalism [22].

Based on the the analysis for the different sector of gauged $\mathcal{N} = 8$ supergravity theory found recently, let us describe the $\text{U}(2) \times \text{U}(2)$ Chern-Simons matter theory. From the superpotential [8] of $\text{U}(2) \times \text{U}(2)$ Chern-Simons matter theory, the quadratic mass deformations are added as follows with the notation of [23]:

$$\frac{T^{-4}}{4!}\epsilon_{ABCD}\text{Tr}\mathcal{Z}^A\mathcal{Z}^{\dagger B}\mathcal{Z}^C\mathcal{Z}^{\dagger D} - 2m^2T^{-2}\text{Tr}\mathcal{Z}^3\mathcal{Z}^{\dagger 3} - 2m^2T^{-2}\text{Tr}\mathcal{Z}^4\mathcal{Z}^{\dagger 4},$$

where \mathcal{Z}^A is also an $\mathcal{N} = 2$ chiral superfield with $\text{SU}(4)$ index $A = 1, 2, \dots, 4$, an operation \dagger is defined by $\mathcal{Z}^{\dagger A} \equiv -i\sigma_2(\mathcal{Z}^A)^T i\sigma_2$ and the T^2 is a monopole operator. The independent two $\text{SU}(2)$ transformations, acting on the first two components of a complex vector made by a linear combination of two components of $\text{SO}(8)$ vector and on the last two components of a complex vector respectively and the overall phase rotation gives a manifest $[\text{SU}(2) \times \text{SU}(2)]_R \times \text{U}(1)$ symmetry. When the undeformed superpotential above is written as $\frac{1}{4}\epsilon_{AC}\epsilon^{BD}\text{Tr}\mathcal{Z}^A\mathcal{W}_B\mathcal{Z}^C\mathcal{W}_D$, the $[\text{SU}(2) \times \text{SU}(2)]_R \times \text{U}(1)$ symmetry is manifest. The relation between the deformed Lagrangian from BL theory and $\mathcal{N} = 2$ superspace description is evident if we integrate the superpotential over the fermionic coordinates. After the integration over the superspace explicitly, the quartic terms arise as well as the mass terms for the fermions.

$\mathcal{N} = 8$ supersymmetry. Let us consider the BL theory with $SO(4)$ gauge group and matter fields. The variation for the bosonic mass term (3.5) plus the fermionic mass term

$$\mathcal{L}_{f.m.} = -\frac{i}{2}h_{ab}\bar{\Psi}^a m\Gamma^{3456}\Psi^b,$$

leads to the following variation

$$\delta\mathcal{L} = ih_{ab}X_I^a(m^2)_{IJ}\bar{\Psi}^b\Gamma_J\epsilon - ih_{ab}\bar{\Psi}^a(m\Gamma^{3456})^2 X_I^b\Gamma_I\epsilon.$$

Then the bosonic mass term $(m^2)_{IJ}\Gamma_J$ should take the form $m^2\sum_{I=3}^{10}\Gamma_I$. Then the diagonal bosonic mass term has nonzero components for all eight elements. Then one obtains the bosonic mass term which appears in (3.5)

$$(m^2)_{IJ} = \text{diag}(m^2, m^2, m^2, m^2, m^2, m^2, m^2, m^2).$$

Once again we describe the different sector of gauged $\mathcal{N} = 8$ supergravity found recently. From the superpotential of $U(2) \times U(2)$ Chern-Simons matter theory, the quadratic mass deformations in this case are added as follows [23]:

$$\frac{T^{-4}}{4!}\epsilon_{ABCD}\text{Tr}\mathcal{Z}^A\mathcal{Z}^{\dagger B}\mathcal{Z}^C\mathcal{Z}^{\dagger D} - \frac{m^2}{2}T^{-2}\sum_{A=1}^4\text{Tr}\mathcal{Z}^A\mathcal{Z}^{\dagger A}.$$

There exists a manifest $SU(2) \times SU(2) \times U(1) \times \mathbf{Z}_2$ symmetry. After the integration over the superspace, one sees that there are also quartic terms. See [23] for more details.

Let us describe the case where the supersymmetry is lower than $\mathcal{N} = 1 G_2$ invariant case.

$\mathcal{N} = \frac{1}{2}$ supersymmetry. We have to introduce more mass parameters within the structure of (3.1). Let us consider the following fermionic mass terms

$$\begin{aligned} \mathcal{L}_{f.m.} = & -\frac{i}{2}h_{ab}\bar{\Psi}^a(m_1\Gamma^{78910} + m_2\Gamma^{56910} - m_3\Gamma^{5678} + m_4\Gamma^{46810} + m_5\Gamma^{4679} + m_6\Gamma^{4589} \\ & - m_7\Gamma^{45710} - m_8\mathbf{1} - n_1\Gamma^{3456} - n_2\Gamma^{3478} + n_3\Gamma^{34910} - n_4\Gamma^{3579} - n_5\Gamma^{35810} \\ & - n_6\Gamma^{36710} + n_7\Gamma^{3689})\Psi^b \end{aligned}$$

where the last seven terms are added newly. Now all the fourteen terms of (3.1) are present here. The corresponding fermionic supersymmetry transformation is

$$\begin{aligned} \delta_m\Psi^a = & (m_1\Gamma^{78910} + m_2\Gamma^{56910} - m_3\Gamma^{5678} + m_4\Gamma^{46810} + m_5\Gamma^{4679} + m_6\Gamma^{4589} \\ & - m_7\Gamma^{45710} - m_8\mathbf{1} - n_1\Gamma^{3456} - n_2\Gamma^{3478} + n_3\Gamma^{34910} - n_4\Gamma^{3579} \\ & - n_5\Gamma^{35810} - n_6\Gamma^{36710} + n_7\Gamma^{3689})X_I^a\Gamma_I\epsilon. \end{aligned}$$

We impose the additional seven constraints on the ϵ parameter as well as the previous seven condition (3.4)

$$\begin{aligned} \Gamma^{5678}\epsilon = \Gamma^{56910}\epsilon = \Gamma^{78910}\epsilon = \Gamma^{46810}\epsilon = -\Gamma^{4679}\epsilon = -\Gamma^{4589}\epsilon = -\Gamma^{45710}\epsilon = \\ \Gamma^{34910}\epsilon = \Gamma^{3478}\epsilon = \Gamma^{3456}\epsilon = -\Gamma^{35810}\epsilon = -\Gamma^{36710}\epsilon = -\Gamma^{3689}\epsilon = \Gamma^{3579}\epsilon = -\epsilon. \end{aligned} \quad (3.9)$$

Then it turns out this gives rise to $\frac{1}{16}$ BPS condition(the number of supersymmetry is one). The bosonic mass term $(m^2)_{IJ}\Gamma_J$ should take the form

$$\begin{aligned} & (m_1 + m_2 - m_3 + m_4 - m_5 - m_6 + m_7 - m_8 + n_1 + n_2 - n_3 + n_4 - n_5 - n_6 + n_7)^2 \Gamma_3 \\ & + (m_1 + m_2 - m_3 - m_4 + m_5 + m_6 - m_7 - m_8 + n_1 + n_2 - n_3 - n_4 + n_5 + n_6 - n_7)^2 \Gamma_4 \\ & + (m_1 - m_2 + m_3 + m_4 - m_5 + m_6 - m_7 - m_8 + n_1 - n_2 + n_3 + n_4 - n_5 + n_6 - n_7)^2 \Gamma_5 \\ & + (m_1 - m_2 + m_3 - m_4 + m_5 - m_6 + m_7 - m_8 + n_1 - n_2 + n_3 - n_4 + n_5 - n_6 + n_7)^2 \Gamma_6 \\ & + (m_1 - m_2 - m_3 - m_4 - m_5 + m_6 + m_7 + m_8 + n_1 - n_2 - n_3 - n_4 - n_5 + n_6 + n_7)^2 \Gamma_7 \\ & + (m_1 - m_2 - m_3 + m_4 + m_5 - m_6 - m_7 + m_8 + n_1 - n_2 - n_3 + n_4 + n_5 - n_6 - n_7)^2 \Gamma_8 \\ & + (m_1 + m_2 + m_3 - m_4 - m_5 - m_6 - m_7 + m_8 + n_1 + n_2 + n_3 - n_4 - n_5 - n_6 - n_7)^2 \Gamma_9 \\ & + (m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8 + n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7)^2 \Gamma_{10}. \end{aligned}$$

By taking

$$\begin{aligned} m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 &= \frac{m_8}{2} = \\ n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 &= m, \end{aligned}$$

the diagonal bosonic mass term has nonzero component only for 1010 and other components are vanishing. Then one obtains the bosonic mass term which appears in (3.5)

$$(m^2)_{IJ} = \text{diag}(0, 0, 0, 0, 0, 0, 0, 0, 256m^2).$$

Note that this form looks like $\mathcal{N} = 1$ supersymmetric case because there exists only nonzero component for the last element but the fermionic mass terms are different from each other. It would be interesting to construct the gravity dual for this $\mathcal{N} = \frac{1}{2}$ mass deformed BL theory.

$\mathcal{N} = 0$ supersymmetry. This can be obtained from (3.9) by changing the signs of the three constraints in the second line of (3.9) as follows:

$$\begin{aligned} \Gamma^{5678}\epsilon &= \Gamma^{56910}\epsilon = \Gamma^{78910}\epsilon = \Gamma^{46810}\epsilon = -\Gamma^{4679}\epsilon = -\Gamma^{4589}\epsilon = -\Gamma^{45710}\epsilon = \\ \Gamma^{34910}\epsilon &= \Gamma^{3478}\epsilon = \Gamma^{3456}\epsilon = \Gamma^{35810}\epsilon = \Gamma^{36710}\epsilon = \Gamma^{3689}\epsilon = \Gamma^{3579}\epsilon = -\epsilon. \end{aligned}$$

The bosonic mass terms take the form similarly as above and by taking

$$\begin{aligned} m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 &= \frac{m_8}{2} = \\ n_1 = n_2 = n_3 = n_4 = -n_5 = -n_6 = -n_7 &= m, \end{aligned}$$

the diagonal bosonic mass term has nonzero component only for 1010 and other components are vanishing. Then one obtains the bosonic mass term which appears in (3.5)

$$(m^2)_{IJ} = \text{diag}(0, 0, 0, 0, 0, 0, 0, 0, 256m^2). \tag{3.10}$$

We can integrate out the massive scalar Φ_8 with adjoint index at a low enough scale and this results in the sixth order superpotential $\text{Tr}(Y^{\pm ij k 8} \Phi_i \Phi_j \Phi_k)^2 +$

$\text{Tr } \epsilon_{ijklmnp} Y^{\pm ij k 8} (\Phi_l \Phi_m \Phi_n \Phi_p)$ with upper sign for $\text{SO}(7)^+$ case and lower one for $\text{SO}(7)^-$ case in terms of $\mathcal{N} = 1$ superfields.

For different parametrizations

$$\begin{aligned} m_1 = m_2 = m_3 = m_4 = m_5 = m_6 = m_7 = \\ -\frac{n_1}{2} = -n_2 = -n_3 = -n_4 = n_5 = n_6 = n_7 = m, \quad m_8 = 0, \end{aligned}$$

the diagonal bosonic mass term has nonzero eight components. Then one obtains the bosonic mass term which appears in (3.5)

$$(m^2)_{IJ} = \text{diag}(m^2, m^2, m^2, m^2, m^2, m^2, m^2, m^2). \quad (3.11)$$

Then the full superpotential is given by the eight mass terms coming from (3.11) and quartic terms from $f_{abcd} Y^{\pm ijkl} \text{Tr } \Phi_i^a \Phi_j^b \Phi_k^c \Phi_l^d$ that are necessary to the original $\mathcal{N} = 8$ supersymmetry before mass deformation. We turn on the mass perturbation in the UV and flow to the IR. This maps to turning on certain fields in the AdS_4 supergravity where they approach to zero in the UV and develop a nontrivial profile as a function of r as one goes to the IR.

4 The potential and metric on the moduli space of an M2-brane probe

The eleven-dimensional metric with the warp factor can be written as [24]

$$ds_{11}^2 = ds_4^2 + ds_7^2 = \Delta(x, y)^{-1} g_{\mu\nu}(x) dx^\mu dx^\nu + G_{mn}(x, y) dy^m dy^n, \quad (4.1)$$

where the warp factor $\Delta(x, y)$ depends on both the four-dimensional spacetime x^μ ($\mu = 1, 2, 3, 4$) and seven-dimensional internal space y^m ($m = 1, 2, \dots, 7$). The four-dimensional metric which has a three-dimensional Poincare invariance takes the form $g_{\mu\nu}(x) dx^\mu dx^\nu = e^{2A(r)} \eta_{\mu'\nu'} dx^{\mu'} dx^{\nu'} + dr^2$, where $\eta_{\mu'\nu'} = (-, +, +)$ and $r \equiv x^4$ is the coordinate transverse to the domain wall as in section 2 and the scale factor $A(r)$ behaves linearly in r at UV and IR regions. The metric formula by [24] generates the 7-dimensional metric $G_{mn}(x, y)$ from the four input data of AdS_4 vacuum expectation values for scalar and pseudo-scalar fields $(\lambda, \lambda', \alpha, \phi)$. We'll use the different parametrizations instead of using these fields directly.

Let us introduce the redefined fields as follows:

$$\begin{aligned} a &\equiv \cosh\left(\frac{\lambda}{\sqrt{2}}\right) + \cos \alpha \sinh\left(\frac{\lambda}{\sqrt{2}}\right), \\ b &\equiv \cosh\left(\frac{\lambda}{\sqrt{2}}\right) - \cos \alpha \sinh\left(\frac{\lambda}{\sqrt{2}}\right), \\ c &\equiv \cosh\left(\frac{\lambda'}{\sqrt{2}}\right) + \cos \phi \sinh\left(\frac{\lambda'}{\sqrt{2}}\right), \\ d &\equiv \cosh\left(\frac{\lambda'}{\sqrt{2}}\right) - \cos \phi \sinh\left(\frac{\lambda'}{\sqrt{2}}\right). \end{aligned} \quad (4.2)$$

The seven-dimensional metric turns out to be

$$ds_7^2 \equiv G_{mn} dy^m dy^n = \sqrt{\Delta} \left(\frac{a^5 b^5 d^3}{c^3} \right)^{\frac{1}{4}} L^2 \sum_{i=1}^7 e^i \otimes e^i, \quad (4.3)$$

where e^i is the local frames in [10], L is a radius of round seven-sphere, and the warp factor is determined as

$$\Delta = \left(\frac{ab}{cd}\right)^{\frac{1}{6}} c^{-1} \xi^{-\frac{4}{3}}. \quad (4.4)$$

Here the $SU(3)$ invariant deformed norm on the seven-sphere, from the deformation matrix, becomes

$$\xi^2 = \eta \cos^2 \mu + (\eta_1 \cos^2 \psi + \eta_2 \sin^2 \psi) \sin^2 \mu \quad (4.5)$$

with deformation parameters

$$\eta = \left(\frac{a^3 b^5 d^3}{c^3}\right)^{\frac{1}{4}}, \quad \eta_1 = \left(\frac{a^7 bc}{d}\right)^{\frac{1}{4}}, \quad \eta_2 = \left(\frac{a^7 bd^7}{c^7}\right)^{\frac{1}{4}}. \quad (4.6)$$

At the $SO(8)$ fixed point where $\eta = 1 = \eta_1 = \eta_2$ because of $a = b = c = d = 1$, ξ^2 becomes 1. Note that the two coordinates among eight coordinates are parametrized by

$$X_7 = \sin \mu \cos \psi \quad \text{and} \quad X_8 = \sin \mu \sin \psi. \quad (4.7)$$

Recall that \mathbf{S}^1 of $U(1)$ Hopf fiber on \mathbf{CP}^3 is embedded in \mathbf{R}^2 spanned by X_7 and X_8 and the \mathbf{S}^5 given by Hopf fibration on \mathbf{CP}^2 is embedded in \mathbf{R}^6 spanned by $(X_1, X_2, X_3, X_4, X_5, X_6)$.

Let us describe the $\mathcal{N} = 8$ four-dimensional gauged supergravity. Now we go to the $SL(8, R)$ basis [25] and introduce the rotated vielbeins

$$\begin{aligned} U_{IJ}^{ij} &= u_{ab}^{ij} (\Gamma_{IJ})^{ab}, & V^{ijIJ} &= v^{ijab} (\Gamma_{IJ})^{ab} \\ U_{ij}^{IJ} &= u_{ij}^{ab} (\Gamma_{IJ})^{ab}, & V_{ijIJ} &= v_{ijab} (\Gamma_{IJ})^{ab} \end{aligned}$$

where all indices i, j and a, b run from 1 to 8 and correspond to the realization of $E_{7(7)}$ in the $SU(8)$ basis and Γ_{IJ} are the $SO(8)$ generators in [3]. We also define the following quantities

$$\begin{aligned} A_{ijIJ} &= \frac{1}{\sqrt{2}} (U_{ij}^{IJ} + V_{ijIJ}), & B_{ij}^{IJ} &= \frac{1}{\sqrt{2}} (U_{ij}^{IJ} - V_{ijIJ}), \\ C_{IJ}^{ij} &= \frac{1}{\sqrt{2}} (U_{IJ}^{ij} + V^{ijIJ}), & D^{ijIJ} &= \frac{1}{\sqrt{2}} (-U_{IJ}^{ij} + V^{ijIJ}). \end{aligned}$$

Then the ‘‘geometric’’ T tensor [25] can be written as

$$\tilde{T}_l^{kij} = \frac{1}{168\sqrt{2}} C_{LM}^{ij} \left(A_{lmJK} D^{kmKI} \delta_I^L x_M x_J - B_{lm}^{JK} C_{KI}^{km} \delta_J^M x_L x_I \right)$$

where we have a relation between x_I and X_I that is a coordinate for $\mathbf{R}^8: \sum_{I=1}^8 (X_I)^2 = 1$:

$$\begin{aligned} X_1 &\equiv \frac{1}{\sqrt{2}}(x_2 - x_6), & X_2 &\equiv -\frac{1}{\sqrt{2}}(x_3 - x_7), \\ X_3 &\equiv \frac{1}{\sqrt{2}}(x_4 - x_8), & X_4 &\equiv -\frac{1}{\sqrt{2}}(x_1 - x_5), \\ X_5 &\equiv \frac{1}{\sqrt{2}}(x_2 + x_6), & X_6 &\equiv \frac{1}{\sqrt{2}}(x_3 + x_7), \\ X_7 &\equiv \frac{1}{\sqrt{2}}(x_4 + x_8), & X_8 &\equiv \frac{1}{\sqrt{2}}(x_1 + x_5). \end{aligned}$$

From this, the corresponding “geometric” A_1 tensor is given by $\tilde{A}_1^{ij} = \tilde{T}_m^{imj}$.

By computing the 88 component of this A_1 tensor, one obtains the geometrical superpotential W_{gs} as follows:

$$\begin{aligned}
 W_{gs}^2 \equiv |\tilde{A}_1^{88}|^2 &= ab^2c^2d^2 + 2ad^2 \left[bc(ad - bc) - 2(\sqrt{ab-1} - \sqrt{cd-1})^2 \right] X_7^2 \\
 &\quad + 2ac^2 \left[bd(ac - bd) - 2(\sqrt{ab-1} + \sqrt{cd-1})^2 \right] X_8^2 \\
 &\quad + 2a \left[2d^2(\sqrt{ab-1} - \sqrt{cd-1})^2 - 2c^2(\sqrt{ab-1} + \sqrt{cd-1})^2 \right. \\
 &\quad \left. + 8a^2(1 - cd) + cd(ad - bc)(ac - bd) + 4c^2(ab + cd - 2) \right] X_7^2 X_8^2 \\
 &\quad + ad^2 \left[(ad - bc)^2 + 4(\sqrt{ab-1} - \sqrt{cd-1})^2 \right] X_7^4 \\
 &\quad + ac^2 \left[(ac - bd)^2 + 4(\sqrt{ab-1} + \sqrt{cd-1})^2 \right] X_8^4. \tag{4.8}
 \end{aligned}$$

When the conditions $d = b$ and $c = a$ (i.e., G_2 -invariant sector which contains $G_2, \text{SO}(7)^\pm$ critical points) are satisfied, this geometric superpotential does not contain X_7 dependence and becomes the expression given in [9]. Furthermore, by calculating the A_2 tensor obtained from the geometric T tensor, one arrives at the geometric scalar potential

$$\begin{aligned}
 V_{gp}(a, b, c, d, X_7, X_8) &= -g^2 \left(\frac{3}{4} |\tilde{A}_1^{ij}|^2 - \frac{1}{24} |\tilde{A}_2^i{}_{jkl}|^2 \right) \\
 &= 8g^2a \left[bcd + d(ad - bc)X_7^2 + c(ac - bd)X_8^2 \right]^2 \tag{4.9}
 \end{aligned}$$

where the constraint equation $\sum_{I=1}^8 (X_I)^2 = 1$ is used several times for obtaining this simple form. Moreover, the expression $|\tilde{A}_1^{77} + \tilde{A}_1^{88}|^2$ with the conditions $a = \frac{1}{b}, c = d$ (i.e., $\text{SU}(3) \times \text{U}(1)$ critical point) leads to $16\tilde{W}^2$ where \tilde{W} is a geometric superpotential found in [4]. When the conditions $d = b$ and $c = a$ (i.e., G_2 -invariant sector) are satisfied, this geometric scalar potential becomes the expression given in [9]. It is obvious that the coefficient of X_7 vanishes in (4.9) at G_2 -invariant sector condition.

When the particular conditions for \mathbf{R}^2 are satisfied

$$X_7 = X_8 = \frac{1}{2\sqrt{2}}, \quad \text{or} \quad \sin \mu = \frac{1}{2}, \quad \text{and} \quad \sin \psi = \frac{1}{\sqrt{2}},$$

the geometric superpotential leads to the superpotential W (that is, $W_{gs} = W$) where the superpotential W introduced in (2.4) can be rewritten, using the relations (4.2), in terms of redefined fields

$$\begin{aligned}
 W^2 &= \frac{a}{64} \left[12ab(-2 + cd)(c^2 + d^2) + a^2(16 + c^4 - 16cd + 2c^2d^2 + d^4) \right. \\
 &\quad \left. - 12(2c^3d + 2cd^3 - 4d^2(1 + \sqrt{(ab-1)(cd-1)}) \right. \\
 &\quad \left. + c^2(-4 - 3b^2d^2 + 4\sqrt{(ab-1)(cd-1)}) \right]. \tag{4.10}
 \end{aligned}$$

In terms of (a, b, c, d) , the flow equations (2.5) read in symmetric form

$$\begin{aligned}
 \partial_r a &= \frac{8}{3L} [a^2 \partial_a W + (ab - 2) \partial_b W], \\
 \partial_r b &= \frac{8}{3L} [(ab - 2) \partial_a W + b^2 \partial_b W], \\
 \partial_r c &= \frac{2}{L} [c^2 \partial_c W + (cd - 2) \partial_d W], \\
 \partial_r d &= \frac{2}{L} [(cd - 2) \partial_c W + d^2 \partial_d W], \\
 \partial_r A &= -\frac{2}{L} W,
 \end{aligned} \tag{4.11}$$

where the superpotential is the same as AdS_4 superpotential (2.4) but now is given by (4.10). Note that the derivatives of W with respect to a, b, c and d do not vanish at the critical points but the r -derivatives of a, b, c and d do vanish at the critical point because they have the factors $\frac{d\lambda}{dr}$, $\frac{d\lambda'}{dr}$, $\frac{d\alpha}{dr}$ and $\frac{d\phi}{dr}$ by chain rule. In other words, for example, the full expression in the first equation of (4.11), $a^2 \partial_a W + (ab - 2) \partial_b W$, at the critical point vanishes even though each term $\partial_a W$ and $\partial_b W$ does not vanish at the critical point.

The action for the M2-brane probe has two pieces, DBI term and WZ term, and it contains the pull back metric and three-form onto the M2-brane. We consider a probe that is parallel to the source M2-branes and it is traveling at a small velocity transverse to its world volume. This leads to a potential and a kinetic term for the M2-brane probe. If the potential vanishes then the kinetic term provides us with a metric on the corresponding moduli space. The potential seen by the M2-brane probe [25–27] has a factor

$$\begin{aligned}
 e^{3A} (\Delta^{-\frac{3}{2}} - W_{gs}) &= e^{3A} \sqrt{abcd} \left[1 + \left(\frac{ac}{bd} - 1 \right) X_7^2 + \left(\frac{ad}{bc} - 1 \right) X_8^2 \right] \\
 &\times \left(1 - \sqrt{1 - \frac{[1 + (\frac{ac}{bd} - 1) X_7^2 + (\frac{ad}{bc} - 1) X_8^2]^2 - \frac{W_{gs}^2}{a^2 b^2 c^2 d^2}}{[1 + (\frac{ac}{bd} - 1) X_7^2 + (\frac{ad}{bc} - 1) X_8^2]^2}} \right)
 \end{aligned} \tag{4.12}$$

where we use (4.4) and (4.8) with the deformed norm $\xi^2 = \eta [1 + (\frac{ac}{bd} - 1) X_7^2 + (\frac{ad}{bc} - 1) X_8^2]$ from (4.5) and (4.6). The moduli spaces of the brane probe are given by the loci where the potential vanishes. One sees that this potential (4.12) vanishes at $X_7 = 0 = X_8$. By realizing that $X_7 = \sin \mu \cos \psi$ and $X_8 = \sin \mu \sin \psi$ appeared in (4.7), this leads to $\mu = 0$. On this subspace, the six-dimensional moduli space from (4.3), by multiplying the factor $e^A \Delta^{-1/2}$ into G_{mn} , is given by

$$ds^2|_{\text{moduli}} = \sqrt{a} \eta L^2 e^A \left(\sum_{i=1}^4 e^i \otimes e^i + e^7 \otimes e^7 \right) + (e^A \sqrt{abcd}) dr^2 \tag{4.13}$$

where the reduced frames are given by

$$\begin{aligned}
 e^1 &= \frac{1}{\sqrt{\eta}} d\theta, & e^2 &= \frac{1}{\sqrt{\eta}} \frac{1}{2} \sin \theta \sigma_1, & e^3 &= \frac{1}{\sqrt{\eta}} \frac{1}{2} \sin \theta \sigma_2, & e^4 &= \frac{1}{\sqrt{\eta}} \frac{1}{2} \sin \theta \cos \theta \sigma_3, \\
 e^7 &= \sqrt{\frac{cd}{\eta ab}} \left[d(\phi + \psi) + \frac{1}{2} \sin^2 \theta \sigma_3 \right]
 \end{aligned}$$

and we used the relations (4.4) and (4.5): $\Delta = \left(\frac{ab}{cd}\right)^{\frac{1}{6}} c^{-1} \eta^{-\frac{2}{3}}$ and $\xi^2 = \eta$. The behavior of (4.13) can be verified by a numerical study of the flow equations for the functions $(a(r), b(r), c(r), d(r), A(r))$. This metric on the moduli space is for SU(3) invariant sector containing six critical points. We have movement on the (squashed) \mathbf{S}^5 with coordinates $(\theta, \alpha_1, \alpha_2, \alpha_3, \phi)$ with three Euler angles α_i and the radial direction r .

As we approach the IR critical point, a new radial coordinate $u \simeq e^{\frac{1}{2}A(r)}$ is introduced. Moreover, using the flow equations (2.5), (2.6) or (2.7), $\partial_r A = \frac{2}{L}W$ with $g \equiv \frac{\sqrt{2}}{L}$, one can express the derivative with respect to r in terms of u . That is, $du^2 = \frac{W^2 u^2}{L^2} dr^2$. By substituting this into (4.13), the six-dimensional moduli space transverse to M2-branes can be written as

$$ds^2|_{\text{moduli}} = \frac{L^2 \sqrt{abcd}}{W^2} \left(du^2 + \frac{W^2}{bcd} u^2 ds_{FS(2)}^2 + \frac{W^2}{ab^2} u^2 \left[d(\phi + \psi) + \frac{1}{2} \sin^2 \theta \sigma_3 \right]^2 \right) \quad (4.14)$$

where the Fubini-Study metric on $\mathbf{CP}^2 (\simeq \mathbf{S}^4$ when the angle θ is very small, i.e., $\cos \theta \simeq 1$) is given by $ds_{FS(2)}^2 = d\theta^2 + \frac{1}{4} \sin^2 \theta (\sigma_1^2 + \sigma_2^2 + \cos^2 \theta \sigma_3^2)$ and $[d(\phi + \psi) + \frac{1}{2} \sin^2 \theta \sigma_3]$ is the Hopf fiber on it. The η dependence on the reduced frames e^1, e^2, e^3 and e^4 has been cancelled by the overall factor η in (4.13). Therefore, there is no deformation within $ds_{FS(2)}^2$. At the UV end, SO(8) fixed point where $a = 1 = b = c = d = W$, the metric becomes $ds^2|_{\text{moduli}} \sim du^2 + u^2 d\Omega_5^2$. Here $d\Omega_5^2$ is the \mathbf{S}^5 metric given by Hopf fibration on \mathbf{CP}^2 base. Moreover, for $X_7 = 0 = X_8$, the three-form potential becomes $A^{(3)} \sim H^{-1} dt \wedge dx^1 \wedge dx^2$ where the function $H = e^{-3A} \Delta^{3/2}$.

At the IR end of nonsupersymmetric SO(7)⁺ flow, by inserting the critical values

$$a = 5^{\frac{1}{4}} = c, \quad b = 5^{-\frac{1}{4}} = d, \quad W = \frac{3}{2} \cdot 5^{-\frac{1}{8}} \quad : \quad \text{SO}(7)^+ \text{ symmetry}$$

into (4.14), one reads off the coefficients of the second, third terms of (4.14) ($\frac{W^2}{bcd}$ and $\frac{W^2}{ab^2}$) as $\frac{9}{4}$ and $\frac{9}{4}$ respectively. The mass spectrum for the $\frac{\sqrt{7}}{2}\lambda$ around SO(7)⁺ fixed point can be computed as in [28] and it is given by 6. At the IR end of the flow, $A(r) \sim \frac{3 \cdot 5^{-\frac{1}{8}}}{L} r$ and $u \sim e^{\frac{3 \cdot 5^{-\frac{1}{8}}}{2L} r} \sim e^{\frac{A(r)}{2}}$ above. Then the superfield S becomes $S = (\Phi_1^2 + \dots + \Phi_7^2)^{\frac{9}{4}}$ in the boundary theory. Note that the mass-deformed bosonic terms are characterized by (3.10). The power $\frac{9}{4}$ comes from the factor in the metric of the moduli. From the tensor product between $\mathbf{7}$ and $\mathbf{7}$ of SO(7)⁺ representation, one gets a singlet $\mathbf{1}$. For the superfield $S(x, \theta)$, the action looks like $\int d^3x d^2\theta S(x, \theta)$. This implies that the highest component field in θ -expansion has a conformal dimension 7 in the IR.

At the IR end of nonsupersymmetric SO(7)⁻ flow, by inserting the critical values

$$a = \frac{\sqrt{5}}{2} = c, \quad b = \frac{\sqrt{5}}{2} = d, \quad W = \frac{3}{8} \cdot 5^{\frac{3}{4}} \quad : \quad \text{SO}(7)^- \text{ symmetry}$$

into (4.14), one reads off the coefficients of the second, third terms of (4.14) as $\frac{9}{8}$ and $\frac{9}{8}$ respectively. The mass spectrum for the $\frac{\sqrt{7}}{2}\lambda$ around SO(7)⁻ fixed point is given by 6 also. At the IR end of the flow, $A(r) \sim \frac{3 \cdot 5^{\frac{3}{4}}}{4L} r$ and $u \sim e^{\frac{3 \cdot 5^{\frac{3}{4}}}{2L} r} \sim e^{\frac{A(r)}{2}}$ above. Then the

superfield S is given by $S = (\Phi_1^2 + \dots + \Phi_7^2)^{\frac{9}{8}}$ in the boundary theory. From the tensor product between $\mathbf{7}$ and $\mathbf{7}$ of $SO(7)^-$ representation, one gets a singlet $\mathbf{1}$. The highest component field in θ -expansion has a conformal dimension 7 in the IR.

At the IR end of supersymmetric $SU(3) \times U(1)$ flow, by inserting the critical values

$$a = \sqrt{3} = \frac{1}{b}, \quad c = \sqrt{\frac{3}{2}} = d, \quad W = \frac{1}{2} \cdot 3^{\frac{3}{4}} \quad : \quad SU(3) \times U(1) \text{ symmetry}$$

into (4.14), one reads off the coefficients of the second, third terms of (4.14) as $\frac{3}{2}$ and $\frac{9}{4} = (\frac{3}{2})^2$ respectively [4]: stretching factors. The form of IR metric is a consequence of the power $\frac{3}{2}$ of u^2 which appears in the Kahler potential [27]. There is a conical singularity at the origin. The S^5 's in constant r or u radial slices are squashed by the presence of $\frac{cd}{ab}$ (which is equal to $\frac{3}{2}$ in the IR) instead of unity, after taking out the common factor $\frac{W^2}{bcd}u^2$ inside of the bracket in (4.14). The residual isometry or the global symmetry of boundary field theory provides this deformation. Note that the values $a = 1 = \frac{1}{b}$ and $c = \sqrt{2} = d$ lead to the nonsupersymmetric $SU(4)^-$ critical point.

Finally, at the IR end of supersymmetric G_2 flow, by inserting the critical values

$$a = \sqrt{\frac{6\sqrt{3}}{5}} = c, \quad b = \sqrt{\frac{2\sqrt{3}}{5}} = d, \quad W = \sqrt{\frac{36\sqrt{2} \cdot 3^{\frac{1}{4}}}{25\sqrt{5}}} \quad : \quad G_2 \text{ symmetry}$$

into (4.14), one reads off the coefficients of the second, third terms of (4.14) as $\frac{6}{5}$ and $\frac{6}{5}$ respectively [9].

Contrary to the $\mathcal{N} = 2$ $SU(3) \times U(1)$ supersymmetric flow, for the $SO(7)^\pm$ and G_2 critical points, the squashing parameter $\frac{cd}{ab}$ inside of S^5 becomes one due to the fact that $d = b$ and $c = a$. The metric looks like $ds^2|_{\text{moduli}} \sim du^2 + \left(\frac{W^2}{bcd}\right) u^2 d\Omega_5^2$. The values $\frac{9}{4}, \frac{9}{8}$ and $\frac{6}{5}$ coming from $\frac{W^2}{bcd}$ arise as above. In spite of their difference, both sectors, $SU(3) \times U(1)$ invariant sector and G_2 invariant sector share the same CP^2 as mentioned before.

5 Conclusions and outlook

We have found the two nonsupersymmetric flow equations preserving $SO(7)^\pm$, presented its dual theories by adding the mass terms and analyzed the M2-brane analysis of moduli space with the IR behaviors. In particular, (4.8) and (4.14) are necessary to perform this analysis.

It is an open problem to find out the eleven-dimensional lift of $SU(3)$ invariant sector using (4.8). The three-form potential looks like $A^{(3)} = -e^{3A(r)}W_{gs}(r, \mu, \psi) + \dots$. We have to solve the eleven-dimensional Einstein-Maxwell equations to complete the eleven-dimensional lift of whole $SU(3)$ -invariant sector including RG flows. The eleven-dimensional metric is given by (4.1) where the compact 7-manifold metric G_{mn} and the warp factor Δ are completely determined by (4.3) and (4.4) in the local frames. The geometric parameters $a(r), b(r), c(r), d(r)$ depend on the AdS_4 radial coordinate r and are subject to the RG flow equations (4.11) in 4-dimensional gauged supergravity. The local

frame is useful to achieve this work. As performed in [4], one easily makes an ansatz for the 3-form gauge field by using the local frames.

Is there any supersymmetric or nonsupersymmetric flow from $\mathcal{N} = 1$ G_2 to $\mathcal{N} = 2$ $SU(3) \times U(1)$? The negativity of some mass terms around G_2 fixed point supports this possibility. Moreover, the symmetry breaking $G_2 \rightarrow SU(3)$ occurs naturally and this is also other evidence for this RG flow. If there exists such a flow, then it is interesting to study the behavior of spectral function given in [29].

Although the flow equations around $SU(4)^-$ critical point do not exist, it is still open problem to find the solution for $A(r)$ around IR region by numerically along the line of [30]: the equations of motion for the scalar and metric satisfy second order differential equations, in general. See also recent work on this vacuum [31].

What is the gauged supergravity theory corresponding to $\mathcal{N} = 4$ superconformal Chern-Simons matter theory [32]? As mentioned in section 3, due to the symmetry group $SU(2) \times SU(2)$, one needs to search for the $SU(2)$ -invariant sector of the gauged supergravity and some comments on this possibility appeared in [33]. What is the gauged supergravity theory corresponding to $\mathcal{N} = 3$ superconformal Chern-Simons matter theory? For other supergravity with Freund-Rubin compactification, some work is given by [34] and see also recent paper by [35]. For $\mathcal{N} = 1$ case, the similar construction is given in [36]. With $\mathcal{N} = 3$ supersymmetry, since the R symmetry is $SO(3) = SU(2)$, one has to take more singlets among seventy scalars rather than four we considered for $SU(3)$ -invariant sector so far.

What happens when we replace \mathbf{CP}^2 inside the seven-dimensional internal space with $\mathbf{S}^2 \times \mathbf{S}^2$? The eleven-dimensional lift was given in [4]. This can be done by replacing the stretched \mathbf{S}^5 with a space which is topologically $T^{1,1}$. The observation of [4] is that although the eleven-dimensional solutions are different but they do have common four-dimensional gauged $\mathcal{N} = 8$ supergravity. It is an open problem to find an eleven-dimensional metric containing $\mathbf{S}^2 \times \mathbf{S}^2$ with common $SU(3)$ invariance.

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